

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 418 IMAGE PROCESSING

Problem Set 11
Spring 2008

Issued: Thursday, April 10, 2008

Due: Thursday, April 17, 2008

Problem 11.1

In order to better understand the difference between the “spectral scatter” effects of the DFT and DCT, it’s helpful to use a continuous-space analogy. The continuous-space analog of the DFT is the Fourier series.

- (a) What is the Fourier series expansion of the sawtooth waveform: $x(t) = t$ for $0 \leq t < T$, periodic with period T ?
- (b) The continuous-space DCT is the Fourier series of the signal $z(t)$, with period of $2T$, and defined as $z(|t|) = x(t)$ for $-T < t < T$. Demonstrate that the continuous-space DCT of the sawtooth waveform is (a) real-valued, (b) is more lowpass in character than the Fourier series.
- (c) The discrete-space DCT is a bit more complicated than the continuous-space one, because it has to compensate for sampling effects. Consider the image signal:

$$x[n] = n - \frac{N-1}{2}$$

Use symmetry considerations to prove that even terms in the DCT expansion of $x[n]$ are zero.