

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN  
Department of Electrical and Computer Engineering

ECE 418 IMAGE PROCESSING

**Problem Set 1**  
Spring 2008

**Issued:** Thursday, January 17, 2008

**Due:** Thursday, January 24, 2007

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**Problem 1.1**

- (a) Consider the circularly symmetric window function (the “top hat”):

$$w_a(t_1, t_2) = \begin{cases} 1 & t_1^2 + t_2^2 \leq R^2 \\ 0 & \text{else} \end{cases} \quad (1)$$

Is this signal separable?

- (b) Find  $W_a(\Omega_1, \Omega_2)$ , using the definition in Eq. 1.

Recommended solution method: use polar coordinates in both space and frequency:  $t_1 = r \cos \theta$ ,  $t_2 = r \sin \theta$ ,  $dt_1 dt_2 = r dr d\theta$ ,  $\Omega_1 = \rho \cos \phi$ ,  $\Omega_2 = \rho \sin \phi$ . Take advantage of the following identities

$$\cos(\theta - \phi) = \cos \phi \cos \theta + \sin \phi \sin \theta \quad (2)$$

$$J_0(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-jz \sin \theta} d\theta \quad (3)$$

$$y J_1(y) = \int_0^y z J_0(z) dz \quad (4)$$

where  $J_0(z)$  and  $J_1(z)$  are Bessel functions of the first kind. You may find it interesting to know that the “circular sinc” function,  $2J_1(r)/r$ , has many of the same properties as the linear sinc; in particular,  $\lim_{r \rightarrow 0} 2J_1(r)/r = 1$ .

- (c) Consider the discrete-space signal

$$w_d[n_1, n_2] = \begin{cases} 1 & n_1^2 + n_2^2 \leq R^2 \\ 0 & \text{else} \end{cases} \quad (5)$$

Plot  $w_d[n_1, n_2]$  for  $R = 1$ . Is this signal separable? Note that the textbook provides the DSFT  $W_d(\omega_1, \omega_2)$  for  $R = 1$ .

- (d) Plot  $w_d[n_1, n_2]$  (Eq. 5) for  $R = 1.5$ . Is this signal separable? Find  $W_d(\omega_1, \omega_2)$ .

- (e) Is  $W_a(\Omega_1, \Omega_2)$  band-limited? Do you expect that  $W_a\left(\frac{\omega_1}{T_1}, \frac{\omega_2}{T_2}\right) = W_d(\omega_1, \omega_2)$ ? If not, then under what conditions is it true that  $W_a\left(\frac{\omega_1}{T_1}, \frac{\omega_2}{T_2}\right) \approx W_d(\omega_1, \omega_2)$ ?