UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 418 Image Processing

Problem Set 4

Spring 2008

Issued: Thursday, February 6, 2008 **Due:** Thursday, February 13, 2008

Problem 4.1

This problem considers the Frei-Baxter model of color vision, shown on page 61 of the lecture notes.

In this model, the photons impinging on the fovea are sampled by cones. Photons impinge on the fovea with the power spectrum $C(t_1 = n_1T_1, t_2 = n_2T_2, \lambda)$, where t_1 and t_2 are the left-right and up-down positions (measured in degrees of arc), and λ is the wavelength of light (measured in nanometers). The sampling periods are $T_1 = T_2 = 1/60$ degree, meaning that there are about 60 cones of each color in each degree of arc.

Photon absorption is modeled using the activation functions $\alpha_i[n_1, n_2]$, given by

$$\alpha_i[n_1, n_2] = \int S_i(\lambda)C(n_1T_1, n_2T_2, \lambda)d\lambda \tag{1}$$

where $S_1(\lambda)$ is the overall luminance sensitivity curve, shown in Fig. 2.15, but $S_2(\lambda)$ and $S_3(\lambda)$ are the red and blue sensitivity curves, marked γ and α respectively in Fig. 2.17. Notice that the curve in Fig. 2.15 is actually the sum of the three curves in Fig. 2.17.

Fig. 2.19 proposes that the photon absorption rates are log-compressed, then linearly transformed, as follows:

$$c_1[n_1, n_2] = 21.5 \ln \alpha_1[n_1, n_2] \tag{2}$$

$$c_2[n_1, n_2] = 41 (\ln \alpha_1[n_1, n_2] - \ln \alpha_2[n_1, n_2]) \tag{3}$$

$$c_3[n_1, n_2] = 6.27 (\ln \alpha_1[n_1, n_2] - \ln \alpha_3[n_1, n_2]) \tag{4}$$

(a) Suppose that the image is a pure two-color image containing only the colors blue and orange, with spatial distributions specified by $\beta(t_1, t_2)$ and $o(t_1, t_2)$, respectively, thus

$$C(t_1, t_2, \lambda) = \beta(t_1, t_2)\delta(\lambda - 420) + o(t_1, t_2)\delta(\lambda - 650) \tag{5}$$

Express $c_1[n_1, n_2]$ as a function of $\beta(t_1, t_2)$ and $o(t_1, t_2)$. Make appropriate assumptions about the levels in Fig. 2.15.

(b) Using the assumption in Eq. 5, express $c_2[n_1, n_2]$ and $c_3[n_1, n_2]$ as functions of the blue-orange ratio $\rho(t_1, t_2) = \beta(t_1, t_2)/o(t_1, t_2)$. Make appropriate assumptions about the levels in Fig. 2.17.

(c) The chromatic channels $c_2[n_1, n_2]$ and $c_3[n_1, n_2]$ are averaged across a large number of neighboring cones. As a simple model for this averaging process, consider the following filter:

$$B_2[n_1, n_2] = c_2[n_1, n_2] * *h_c[n_1, n_2] \quad (6)$$
$$B_3[n_1, n_2] = c_3[n_1, n_2] * *h_c[n_1, n_2] \quad (7)$$

$$h_c[n_1, n_2] = g_c[n_1]g_c[n_2] \quad (8)$$

where

$$g_c[n] = \begin{cases} 1 & |n| \leq \frac{N-1}{2} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Compute $H_c(\omega_1, \omega_2)$, the DSFT of $h_c[n_1, n_2]$.

(d) Assume that $N = 7$ samples. Plot $G_c(2\pi f_1 T_1)$, the DSFT of the filter specified in Eq. 9, as a function of the spatial frequency f_1. Notice that f_1 is measured in cycles per degree of arc, therefore your plot should have the same axis labels as Fig. 2.20 in the lecture notes (but you may assume linear axis scaling, rather than the logarithmic axis scaling shown in Fig. 2.20)

(e) The Frei-Baxter model proposes that luminance information, $c_1[n_1, n_2]$, is subjected to lateral inhibition. Suppose that lateral inhibition in the fovea is implemented by the following simplified digital filter:

$$B_1[n_1, n_2] = c_1[n_1, n_2] * *h_y[n_1, n_2] \quad (10)$$
$$(11)$$

$$h_y[n_1, n_2] = g_y[n_1]g_y[n_2] \quad (12)$$

where

$$g_y[n] = \begin{cases} 1 & |n| \leq \frac{N_+-1}{2} \\ -\left(\frac{N_+}{N}\right) & \frac{N_+-1}{2} < |n| \leq \frac{N-1}{2} \end{cases} \quad (13)$$

Compute the DSFT, $H_y(\omega_1, \omega_2)$.

(f) Assume that $N = 7$ and $N_+ = 3$. Sketch $G_y(2\pi f_1 T_1)$, the DSFT of the filter described in Eq. 13, as a function of the spatial frequencies f_1. Notice that f_1 is measured in cycles per degree of arc, therefore your plot should have the same axis labels as Fig. 2.20 in the notes (but you may assume linear axis scaling, rather than the logarithmic axis scaling shown in Fig. 2.20).