

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN  
Department of Electrical and Computer Engineering

ECE 418 IMAGE PROCESSING

**Problem Set 8**  
Spring 2008

**Issued:** Thursday, March 13, 2008

**Due:** Thursday, March 27, 2008

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**Problem 8.1**

Suppose that you hire your brother to film the debut performance of your band. Unfortunately, your brother is not a very good videographer, so the image is not well focused. Since you've taken 418, you know that defocus blurring can be modeled using a low-pass filter,  $Y(\vec{\omega}) = H(\vec{\omega})X(\vec{\omega})$ , where a simple model of  $h[\vec{n}]$  might be

$$h[\vec{n}] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & w & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- (a) Assume no noise; for what values of  $w$  can you recover  $X(\vec{\omega})$  from  $Y(\vec{\omega})$ ?
- (b) Assume no noise; specify the frequency response of the filter,  $G(\vec{\omega})$ , used to construct  $\hat{X}(\vec{\omega}) = G(\vec{\omega})Y(\vec{\omega})$  in order to minimize  $E[|\hat{x}[\vec{n}] - x[\vec{n}]|^2]$ , assuming that  $w$  lies in the range you specified in part (a).
- (c) You notice that  $\hat{X}(\vec{\omega})$  has sharper edges than  $Y(\vec{\omega})$ . Why?
- (d) Unfortunately, you also notice that  $\hat{X}(\vec{\omega})$  contains a lot of image noise. Where might the noise have originated (physically), and why is it more perceptible in  $\hat{X}(\vec{\omega})$  than in  $Y(\vec{\omega})$ ?
- (e) Consider a revised model:  $Y(\vec{\omega}) = H(\vec{\omega})X(\vec{\omega}) + V(\vec{\omega})$ , where  $v[\vec{n}]$  is a zero-mean white noise process with variance  $\sigma_v^2$ . Design a filter  $\hat{X}(\vec{\omega}) = G(\vec{\omega})Y(\vec{\omega})$  in order to minimize the MSE  $E[|\hat{x}[\vec{n}] - x[\vec{n}]|^2]$ . In order to design this filter, what one piece of information do you need that has not been specified anywhere in this problem? Make a reasonable assumption, and sketch the resulting  $G(\vec{\omega})$ .
- (f) Does the filter you designed in part (e) enhance or blur edges? Does your answer depend on the type of edge? In what way? (Note: your answer should depend on the assumption you made in part (e)).