Research Statement for Erin Chambers

Computational topology is an exciting new area which is emerging at the intersection of theoretical computer science and mathematics. Algorithmic techniques in topology are far from new, but the increasing use of computational geometry in many disciplines has led to an explosion of new results in both computer science and mathematics. There is currently great demand for algorithms which can make topological guarantees, as well as great opportunity to apply algorithmic proofs in topology. Problems in computational topology are motivated by a wide variety of areas; applications abound in areas such as shape modeling in graphics, pathfinding in motion planning, geometric and topological modeling for protein docking prediction, encoding low dimensional spans for high dimensional data in statistical analysis, and modeling networks using cell complexes and meshes.

Many topological questions are provably hard or even unsolvable in the most general settings, making it necessary to examine specific instances. My research focuses on several instances in this larger setting of computational topology. In each instance, the goal is to find an interesting topological feature, usually a cycle or path with certain desired properties.

Combinatorial Surfaces

A *combinatorial surface* is a 2-manifold which has a graph embedded on it such that all faces in the embedding are topological disks. In this setting, n denotes the number of vertices and edges in the graph, and g denotes the genus, or number of handles, in the underlying surface. Combinatorial surfaces are a natural setting to work in, since algorithms in this model translate nicely to meshes, which are commonly used in graphics and other areas. It has the additional benefit of making the problems solvable, since even relatively simple problems such as finding shortest paths have exponential or even unbounded complexity in more general settings.

Most of my work in this setting involves finding shortest cycles with desired topological properties. A cycle is *noncontractible* if it cannot be continuously deformed to a point, and *nonseparating* if its removal does not disconnect the surface. Finding either type of cycle is a fundamental primitive for topological algorithms. For example, in geometric modeling, cycles of this type often represent extra handles caused by noise in the data.

I developed an algorithm to compute an implicit representation of all pairs shortest paths in a genus g graph where one endpoint is on a fixed face of the embedding [2]; this generalizes a result by Klein on planar graphs. Our algorithm first computes a shortest path tree, then kinetically updates it by simulating the root moving continuously along the edges of the face. As the root moves, the shortest path distances change continuously, but the combinatorial structure of the shortest path tree changes at discrete events. Our analysis bounds the number of times that any edge is added to the shortest path tree, which implies a total running time of $O(g^2 n \log n)$.

The motivation for designing this data structure was a long series of results on finding the shortest noncontractible or shortest nonseparating cycle. We designed the best known algorithm to compute these cycle, in $O(g^2 n \log n)$ time, using our shortest path tree algorithm. Previous algorithms had running times that were either quadratic in n or exponential in g. Our result improves the running time of many applications, as well, including finding approximate TSP tours and embedding graphs in the plane with few crossings.

A *splitting* cycle is a cycle that is both separating and noncontractible. These cycles are a natural extension of planar separators to high genus graphs for divide and conquer algorithms; a splitting cycle necessarily divides both the genus and the vertices, providing two smaller instances of the graph to recurse on. My coauthors and I proved that finding the shortest splitting cycle is NP-Hard, and then showed that the problem is fixed parameter tractable, finding a $g^{O(g)} n \log n$ time algorithm [8].

In future work, I plan to continue to work on new algorithms for genus g graphs. Recent results compute other structures of interest in a planar setting, such as the maximum s-t flow in a directed graph and good approximations for Steiner trees in planar graphs. I plan to extend these results to genus g graphs, as there is significant recent interest in algorithms on topological graphs.

Homotopic Fréchet Distance

Fréchet distance is a natural measure of similarity between curves. Imagine a person walking along one curve and a dog walking along the other, with a leash stretching between the two. The Fréchet distance is the smallest length necessary for such a leash. More formally, for two curves A and B, the Fréchet distance between them is equal to $\inf_{\alpha,\beta:[0,1]\to[0,1]} \left(\max_{0\leq t\leq 1}\ d(A(\alpha(t)), B(\beta(t)))\right)$, where α and β are reparameterizations of $[0,1]$.

In many metric spaces, such as terrains or Euclidean spaces with obstacles present, the distance between two points is the length of a geodesic. Under the formal definition of Fréchet distance, the leash is allowed to jump discontinuously, such as jumping instantaneously over an obstacle or mountain, as long as the length of the leash changes continuously. This is undesirable in some settings; for example, one application uses the motion of the leash to define a correspondence to morph between the curves, which is not possible if the leash jumps discontinuously over obstacles.

In [3], I introduce a continuity requirement on the motion of the leash so that the leash cannot jump over obstacles. I define the *homotopic Fréchet distance* as the Fréchet distance with this additional continuity requirement, and give a polynomial time algorithm to compute the homotopic Fréchet distance between two polygonal curves in the plane with polygonal obstacles.

For some applications, the homotopic Fréchet distance is a better measure of similarity between curves; for example, in robotics, the two curves being compared may be two motion sequences in the configuration space of a robot system. When the configuration space has obstacle regions, the similarity between the two curves is more accurately measured by the homotopic Fréchet distance, which provides the shortest method to deform between the curves that avoids discontinuity.

I plan to continue to extend this work to more general settings. One promising extension is computing homotopic Fréchet distance between curves on any surface with non-positive curvature, since our proofs rely heavily the plane having zero curvature. Our original motivation for the problem was determining the shortest homotopy between two curves on a manifold; my long term goal is to find algorithms or hardness proofs in this setting.

Rips Complexes

I examined the Rips complex of a set of points in the plane with several colleagues [4, 5]. In this simplicial complex, any $k+1$ points which are pairwise within unit distance of each other define a k dimensional simplex. The main advantage of the Rips complex over other simplicial complexes is that it requires little geometric information; the adjacency graph of the points is enough to compute the entire complex. There is recent interest in using Rips complexes to represent sensor networks and to model the topology of high dimensional data sets such as those extracted from digital photographs.

The *shadow* of a Rips complex is the image of the natural projection back to the plane, where each simplex is mapped to the convex hull of its points. I show that a planar Rips complex and its shadow have isomorphic fundamental groups [4]. One interesting consequence of this result is that the fundamental group of a planar Rips complex is a free group, since the shadow consists of a polygonal region with holes in the plane.

We also examine quasi-Rips complexes, which are a way to model uncertainty or error in connectivity [4]. In this setting, there is an edge for any two nodes with distance less than ϵ and no edge if the distance is greater than ϵ'; if the distance is between ϵ and ϵ', the nodes may or may not have an edge between them. We show that no analog of our isomorphism for Rips complexes can hold, although we can gain some information about the shadow complex given two quasi-Rips complexes with disjoint uncertainty intervals.

I also develop algorithms for planar Rips complexes [5]. I first show that even though there are $O(n^3)$ triangles in a planar Rips complex, the boundary of the shadow complex must have linear complexity, and the shadow itself is the union of only $O(n)$ triangles from the Rips complex. These bounds lead to fast algorithms to test contractibility of a cycle in the Rips complex in $O(m \log n)$ time and to find the shortest contractible cycle in $O(n^2 \log n + mn)$ expected time, where n is the number of Rips vertices and m is the number of edges in the Rips complex. The only comparable previous result computed the first homology group using standard Gaussian elimination techniques in $O(n^2 m^2)$ time.

Our algorithmic work on Rips complexes depends entirely on knowing the exact coordinates of the Rips vertices; I plan to determine what can be done algorithmically if no coordinates are known. I also plan to extend our homotopy result to more general settings. For example, consider forming the Rips complex of points sampled from a 2-manifold. Latschev showed that for any closed Riemannian manifold X and ϵ sufficiently small, the Rips complex of any ϵ-sample of X is homotopy equivalent to X. In comparison, our theorem about the shadow is weaker in that it only gives an isomorphism between fundamental groups, but stronger in that it does not require any sampling guarantees. I plan to examine what other types of topological guarantees can be obtained given either stronger or weaker sampling conditions, and how quickly interesting cycles can be computed in this setting.

Combinatorics

In addition to my thesis work, I have also worked with the combinatorics group in the Mathematics department. Over the course of 3 summers, I have worked several variants of domination [1, 6, 7]. All three results establish upper and lower bounds for these parameters in various classes of graphs. Last summer, I studied gap-free degree sequences, investigating which degree sequences are graphic under the constraint that for a degree sequence $d = \{d_1 \dots d_n\}$, d_i is either $d_i + 1$ or $d_{i+1} - 1$ for every $i > 1$. This result will also be submitted over the course of the next year.

While graph theory is not my primary research area, it is frequently useful in almost all areas of algorithms study, and it provides a very challenging and beautiful set of problems which can be made accessible for undergraduates. I plan to continue my own research into problems in this area, hopefully in conjunction my students as well as with faculty and students from the math department.

References

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